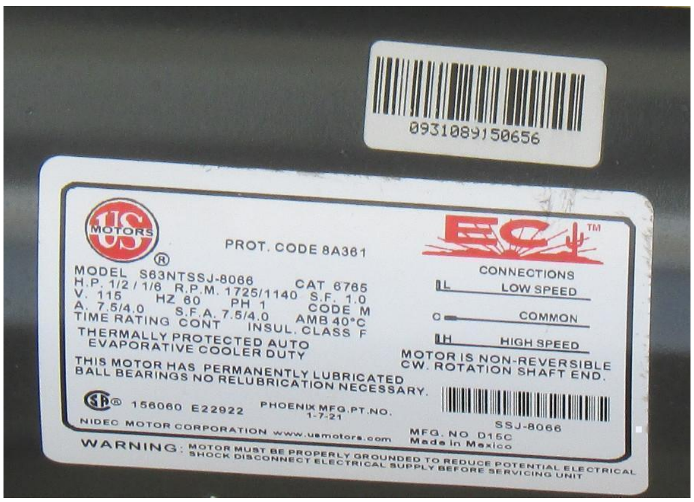
Question:

> A neighbor recently asked me if his evaporative air conditioner was running correctly because he felt the blower drive motor and it was too hot to leave his hand on it for long. I was surprised to find nothing helpful to make a back of the envelope approximation of the operating temperature (the motor casing) of an AC induction motor commonly used as the centrifugal blower drive motor (via belt to pulley on the squirrel cage blower) inside the "swamp coolers" commonly used in the US Southwest states (with dry enough weather to make evaporative cooling effective).

Answer:

1. Regarding the tactile perception of temperature, e.g., “too hot to leave his hand on it for long,” 44° C is considered painful to the touch (per, for example, “Chapter 16 - Heating and Cooling of Buildings; Heat and Mass Transfer: Fundamentals and Applications; ISBN: 0073398187; Copyright year: 2015; http://highered.mheducation.com/sites/dl/free/0073398187/835451/Chapter16.pdf, and the burn threshold of human skin is about 65° C (140° F or 60° C tap water is considered to be “scalding”), per “A Heat Transfer Textbook; “an introduction to heat and mass transfer oriented toward engineering students;” http://web.mit.edu/lienhard/www/ahttv204.pdf. Leeson Electric Corporation states flatly that you cannot judge whether a motor is within operating range by feeling the motor's surface and that modern motors can easily be 80 – 100 degrees C (176 – 212° F), making it inadvisable to attempt to keep a hand on the surface long enough to discern any temperature difference [http://www.leeson.com/TechnicalInformation/hottopic.html]

2. A good first step is to determine what temperature is permitted according to the manufacturer specification for the particular motor. The motor should have a nameplate with specifications required by law (US). I found it convenient to simply photograph the motor *in situ* rather than attempt to copy the specifications by hand (e.g., the nameplate may be oriented upside down, a complication easily corrected by rotating the image). Here is a photograph of the relevant motor nameplate:



We learn from the nameplate that the motor could not have been dangerously overheated or the “THERMALLY PROTECTED AUTO” automatic thermal protection would have open-circuited the motor, turning it off (once an overheated motor cools, the thermal protection circuit will close, permitting the motor to start again). The motor insulation is “CLASS F,” meaning that the NEMA (National Electrical Manufacturers Association) maximum allowable temperature rise for the motor at full load (service factor “S.F. 1.0,” indicates that only the rated horsepower load is permitted) is 105° C plus the ambient temperature around the motor (which is ambient air entering the AC enclosure through evaporative cooling pads in the present context). The maximum ambient temperature is specified as “AMB 40° C” (which is assumed in the NEMA specification). (See Motor Application Guide for Ventilation Products (FA 11301) http://www.greenheck.com/library/articles/3).

So we now know that (1) if the motor exceeds safe operating temperature it will shut down and (2) it is permitted to rise up to 105° C above the surrounding air temperature. For example, if it was a 32° C (90° F) day, the air pulled into the enclosure over the wet cooling pads might cool to 21° C (70° F), so the motor would be permitted to rise up to 105° C plus the 21° C air inside the AC enclosure for measured motor casing temperature of 126° C (which would feel painfully hot per our initial discussion of the perception of heat). However, you would not expect the motor to get anywhere near the NEMA maximum under normal operating conditions.

3. We know what is the permissible maximum, but what would be the typical normal operating temperature of the motor in this application, assuming proper setup and load (e.g., at least 2 square feet of open window per 1000 CFM “Cubic Feet per Minute” supplied by air conditioner, sufficient voltage, proper pulley belt tension, etc.; refer to http://www.homedepot.com/catalog/pdfImages/9b/9b672e80-d85f-4174-90e1-7bfb5e565cb0.pdf CHAMPION•ESSICK Evaporative Cooler Manual)? The nameplate tells us that the motor provides 1/6 horsepower at the lower of the two speeds (the air conditioner was operating in low speed when my neighbor made his touch test of the motor temperature).

1 electric motor horsepower = 746 watts (power), so this 1/6 horsepower (rated output at the lower speed setting) motor is equivalently providing an output power in watts of:

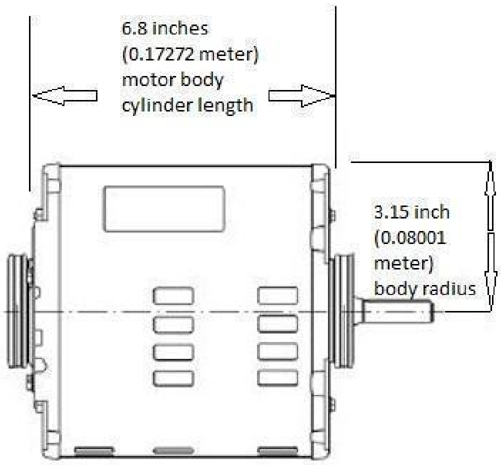
$$\left(\frac{1}{6}\right) horsepower \cdot 746 \frac{watts}{horsepower} = 124.33\, watts \approx 124\, watts$$

The peak efficiency of this type of motor, i.e., a single phase AC induction squirrel cage (specifically a split phase version, which uses a separate start winding out of phase about 30° to develop starting torque, disconnecting via a centrifugal switch once running speed is reached), is 50% to 60% at full load (per Single-Phase AC Induction Squirrel Cage Motors; Twin City Fan Engineering; https://www.tcf.com/docs/fan-engineering-letters/single-phase-ac-induction-squirrel-cage-motors---fe-1100.pdf). In other words, at best about 60% of the electrical energy the motor receives is converted to the mechanical horsepower it delivers at the output shaft. The remainder is lost as heat (which the motor must dissipate to the environment), primarily though winding losses ($heat = I^2 \cdot R$) as current flows in the motor windings), magnetic core losses (induced eddy currents by the rotating magnetic fields of a synchronous AC motor), and mechanical losses from friction (bearings, speaking only of internal motor losses). To obtain 1/6 horsepower (recall that is $\approx 124\, watts$) from a 60% efficient motor you would need to provide input power of:

$$\frac{P_{output}}{\text{Efficiency}} = P_{input} = \frac{124\, watts}{0.60} = 206.7\, watts \approx 207\, watts$$

So $P_{output} - P_{input} = Q_{heat} = 207\, watts - 124\, watts = 83\, watts$, i.e., the equivalent heat transfer rate into the thermodynamic system represented by the motor is 83 watts (we can ignore the work transfer out of this system since that was separately considered).

To calculate a *heat flux* (q), the *heat rate per unit of area*, (and ultimately to estimate temperatures) we need to know the surface area dissipating this Q value. Here is a schematic view of this US Motors catalog number 6765:



Per the US Motor catalog, motor no. 6765 is 9.4" total length with shaft. The shaft is 1.6" so the length is 7.8" including mounting ring hubs. The hubs are 0.5" width so the motor body is only 7.8 – (2 x 0.5) = 6.8 inches = 0.17272 meter. Most of the product specifications for these motors do not give a length other than the total length with shaft, so be forewarned that it is easy to get the body length wrong (as the OP did). The diameter is given as 6.3 inches = 0.16002 meter diameter, so the radius is 0.08001 meter.

The surface area of the body cylinder is $2 \cdot \pi \cdot r \cdot length = 0.086829\, m^2$. The area of the *two* ends of that cylinder is $2 \cdot \pi \cdot r^2 = 0.04022\, m^2$. The entire motor surface is then the sum of the area of the cylinder and the two end caps or $0.086829\, m^2 + 0.04022\, m^2 = 0.127052\, m^2$. The surface heat flux is then $\frac{Q_{watts}}{A_{surface}} = \frac{83\, watts}{0.127052\, m^2} = q_{flux} = 653.28\, \frac{watts}{m^2}$.

Per "A Heat Transfer Textbook" cited earlier the heat flux $q = \bar{h}(T_{body} - T_\infty)$, where $\bar{h}$ is the *heat transfer coefficient* (average over the surface of the body) in units $W/m^2\, K$. This equation (commonly termed "Newton's law of cooling," though Newton never wrote such an expression, but had proposed that the convective cooling of a body would be proportional to $T_{body} - T_\infty$) could also be stated as $Q = \bar{h}A(T_{body} - T_\infty)$, since $q = \frac{Q}{A}$. The surface temperature of the hot (relative to surroundings) body under consideration (the motor casing in present context) is T_{body} and T_∞ is the temperature of the surrounding fluid (air inside the air conditioner in this case) sufficiently far from the surface to constitute free-stream temperature rather than film temperature near the hot surface (see also "Chapter 16... Heat and Mass Transfer," cited above).

I subsequently measured the motor casing and ambient temperature while it was operating at low speed and obtained $T_{body} = 37.77\ ^\circ\text{C}$ (I note as an aside that this would not appear to be painfully hot) and $T_\infty = 15\ ^\circ\text{C}$. We may then calculate the equivalent heat transfer coefficient $\bar{h}$ as (rearranging the equation above relating q and $\bar{h}$):

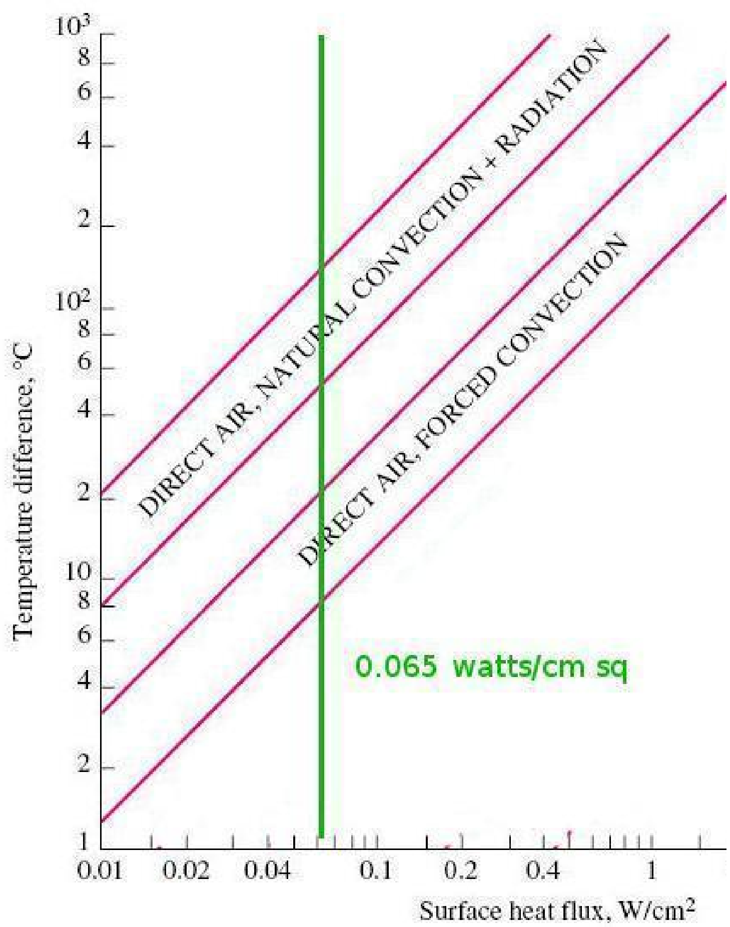
$$\bar{h} = \frac{q}{(T_{body} - T_\infty)} = \frac{653.28\, (watts/m^2)}{37.77\ ^\circ\text{C} - 15\ ^\circ\text{C}} = 28.69\, watts/(m^2 \cdot {}^\circ\text{C})$$

.

By "equivalent heat transfer coefficient," I mean a first approximation *single* transfer estimate that implicitly includes *several* different heat transfer processes. In the present problem we have a hot body (motor) in cooler surroundings (an evaporative air conditioner enclosure). Heat is potentially transferred away from the hot motor to its surroundings by (1) conduction (where it physically is in contact with another surface), (2) natural convection (heated air rising from the motor surface basically), (3) forced convection (air flow as a result of the centrifugal blower fan pulling air into the enclosure), and (4) radiation (heat radiated away from the hot motor surface to the surrounding interior surfaces of the enclosure).

We measured the temperature of the motor and ambient, calculated the likely heat transfer rate Q into the motor (the amount of heat energy it is probably generating) given the efficiency of this class of motor and its operating context here, then calculated the rough overall heat transfer coefficient that would describe the transfer of this heat from the motor (given that the motor was at the temperature we measured). Recall that our question is *what should be the typical operating temperature of the motor* in this context. So we want to analyze the calculated overall or total heat transfer coefficient to decide if it is a proper or expected result and thereby answer our question as to whether the measured temperature of the motor is typical (we know already that the motor is not near the thermal shutdown or insulation breakdown region).

As a rough check on the calculated heat flux and the measured temperature of the motor, we can use a plot of temperature differences ($\Delta T = T_{body} - T_{\infty}$) typical for particular heat flux and heat transfer mechanisms (refer to "Chapter 16... Heat and Mass Transfer" previously cited). Our observed ΔT was 22.77 $^{\circ}$C (37.77 $^{\circ}$C– 15 $^{\circ}$C). Our calculated heat flux was $q = 653.28\, watts/m^2$. The plot uses $watts/cm^2$ rather than $watts/m^2$ so we convert our q value with $653.28\, \frac{watts}{m^2} \cdot \frac{m^2}{10^4 cm^2} = 0.065\, watts/cm^2$. The plot for expected ΔT for forced air convection cooling a $0.065\, watts/cm^2$ surface flux shows an expected range of about $8 - 21\ ^{\circ}$C of possible resulting temperature differences between the hot surface and ambient. The plot for expected ΔT for cooling by direct natural air convection plus radiation for surface flux $0.065\, watts/cm^2$ is about $50 - 120\ ^{\circ}\mathrm{C}$. See below a cropped portion of the plot from "Heat and Mass Transfer," with a green vertical line indicating the $0.065\, watts/cm^2$ reference (follow that line up and then to left horizontally to plot approximate expected ΔT :



Our observed motor temperature, $\Delta T = 22.77\ ^{\circ}\mathrm{C}$, is approximately in the range $8 - 21\ ^{\circ}$C that the plot above indicates is typically possible for forced air cooling of a surface heat flux of $0.065\, watts/cm^2$ so is credible, particularly since we know that our motor has an additional large forced convective process by virtue of the internal wafting of hot air out vents in the motor case.

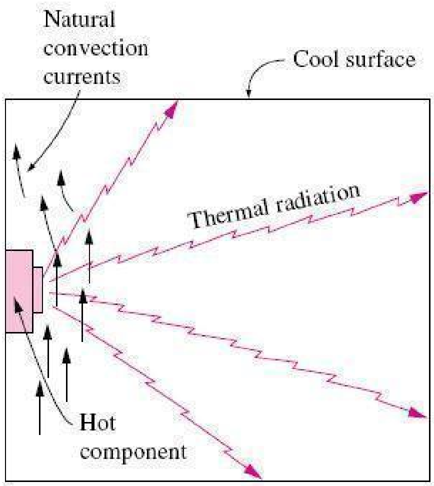
How does our calculated equivalent combined heat transfer coefficient $\bar{h} = 28.69\, watts/(m^2 \cdot {}^{\circ}\mathrm{C})$ compare with values provided in formal thermal analysis papers? For example, "Evolution and Modern Approaches Therm Analysis Electrical Motors" [https://www.scribd.com/document/45479551/Thermal-Analysis-Approaches] states that the combined *natural convection* and *radiation-heat-transfer* coefficient for the equivalent thermal resistance between an AC induction motor external frame and ambient typically lies in the range $12 - 14\, watts/(m^2 \cdot {}^{\circ}\mathrm{C})$. In our case we know that there is external forced convection (or enhanced natural convection from the centrifugal blower intake, the motor being within the upper portion of the adjacent enclosure pad incoming flow) and significant forced convection from within the motor via its internal rotor fin wafting mechanism acting in conjunction with the openings in the motor case to transfer stator and rotor heat directly from the motor to some extent, bypassing the route through the motor casing to ambient. That being said, it seems useful to make the comparison, expecting that the suggested range will be low in comparison: $12 - 14\, watts/(m^2 \cdot {}^{\circ}\mathrm{C})$ vs. $\bar{h} = 28.69\, watts/(m^2 \cdot {}^{\circ}\mathrm{C})$. Our value appears in the expected range as qualified. [For those of you who read the detailed analysis below that follows this basic answer, we will find that without the internal rotor fin wafting heat transfer path this motor has an equivalent combined heat transfer coefficient of about $16\, watts/(m^2 \cdot {}^{\circ}\mathrm{C})$, almost exactly as the expected range, with a slight increase due to the relatively weak forced convective cooling provided within the air conditioner enclosure.]

4. We have answered the original question for the most part, concluding that the operating temperature (the motor casing) of an AC induction motor in a "swamp cooler" is probably going to feel hot to the touch. We quantified this to some extent, estimating human perception of temperature and expected operating temperature of a motor in this application.

For those who are curious, as I was, about the particular processes of heat transfer in this context, I include the following analysis (which may be skipped by those who are satisfied with the answer already provided).

This motor physically contacts only its resilient mounting rings, which are mostly rubber for shock and vibration isolation and do not conduct heat appreciably (nor does the blower fan belt which attaches to the motor pulley).

The magnitude of radiation heat transfer is typically similar in magnitude to that of natural convection. The emittance for a surface painted black is about 0.90. Emittance, ε, is a dimensionless property where $0 < \varepsilon \leq 1$ used to characterize the emissive power of a non-black body. A perfect radiator or black body would have $\varepsilon = 1$. On the other hand, polished unpainted steel is highly reflective and therefore a very poor radiator, having an $\varepsilon \approx 0.07$). Here is a rough figure of a hot component losing heat via natural convection currents and radiation heat transfer to a cooler environment:



For the purposes of a quick estimate of the radiation heat transfer (Q_{rad} in watts) from our motor, we can assume that the motor is completely surrounded by the cooler surfaces of the enclosure, i.e., we will let the view factor be "1" (so we will omit the view factor from the following equation):

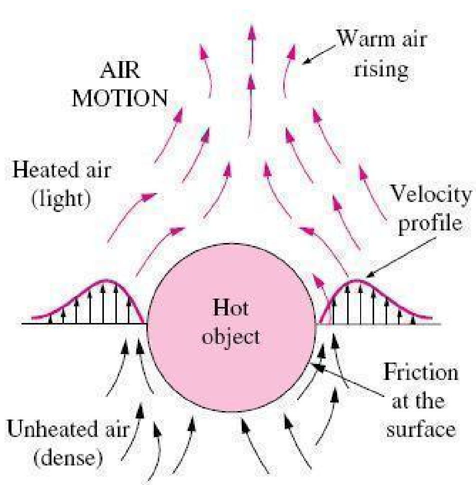
$$Q_{rad} = \varepsilon \cdot A_s \cdot \sigma \cdot (T_{body}^4 - T_{surr}^4)$$

Where ε is the emissivity of the motor surface, 0.90 for black painted metal (we will ignore the fact that our motor end caps are not painted), A_s is the surface area of the motor, $0.127052\, m^2$, and σ is the Stefan-Boltzmann constant, $5.67 \times 10^{-8}\ W/m^2 \cdot K^4$. K = Kelvin degrees and both temperatures in the equation must also be expressed in degrees Kelvin. The temperature of the motor surface is $T_{body} = 37.77\ ^\circ$C and the temperature of the surroundings, T_{surr}, we will approximate by the air temperature inside the air conditioner, $T_{surr} = 15\ ^\circ$C. Converting those temperatures to Kelvin we have $T_{body} =\ 310.92 K^\circ$ and $T_{surr} = 288.15\, K^\circ$. The equation becomes:

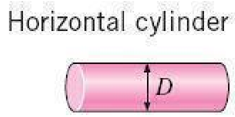
$$Q_{rad} = 0.90 \cdot (0.127052\, m^2) \cdot (5.67 \times 10^{-8}\ W/m^2 \cdot K^4) \cdot (310.92^4 - 288.15^4)$$

We obtain an estimate of $Q_{rad} = 15.89\ watts$ of radiated heat from the motor at the given temperatures. From this radiation heat transfer rate we obtain an equivalent radiation heat transfer coefficient of $h_{rad} = 5.49\, W/m^2 \cdot K$, since $Q = \bar{h} A (T_{body} - T_\infty)$, and we therefore obtain $h_{rad} = \frac{Q_{rad}}{A(T_{body} - T_\infty)}$. We will make use of this radiation heat transfer coefficient later, but for now we note that "Evolution and Modern Approaches Therm Analysis Electrical Motors" (cited earlier) gives $h_{rad} = 5.7\, W/m^2 \cdot K$ as a typical initial estimate of radiation heat transfer coefficient between a motor case and ambient, so our value of $h_{rad} = 5.49\, W/m^2 \cdot K$ appears not unusual.

$Q_{rad} = 15.89\ watts$ is 19% of the 83 watts of heat which must be dissipated by the motor. As we mentioned earlier, the magnitude of radiation heat transfer is typically similar in magnitude to that of natural convection, so let us make a quick estimate of natural convection cooling to compare to the radiation value we just calculated. The following figure illustrates the concept of natural convection here:



Per "Chapter 16... Heat and Mass Transfer" cited earlier, a simplified natural convection heat transfer coefficient relation for a horizontally positioned cylinder (at atmospheric pressure and assumed laminar flow) is:

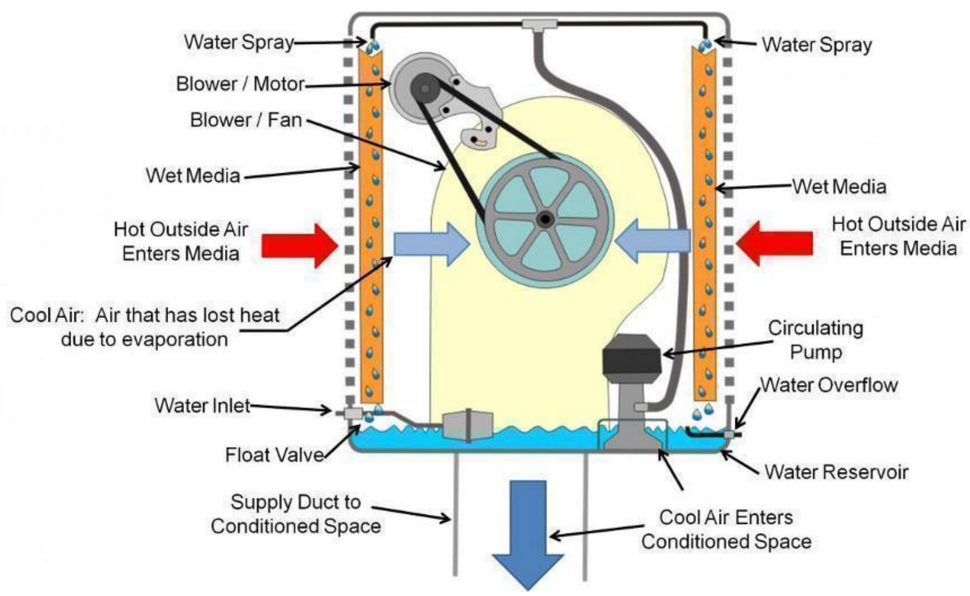


$$h_{\text{conv}} = 1.32\left(\frac{\Delta T}{D}\right)^{0.25}$$

For this equation we use $\Delta T = T_{body} - T_{\infty} = 37.77\ ^{\circ}\mathrm{C} - 15\ ^{\circ}\mathrm{C} = 22.77\ ^{\circ}\mathrm{C}$ and $D = 0.16002\, m$, giving $h_{conv} = 1.32 \cdot (\frac{22.77\ ^{\circ}\mathrm{C}}{0.16002\ m})^{0.25} = 4.5590\, W/(m^2 \cdot\ ^{\circ}\mathrm{C})$. This is an empirically determined relation and the resulting heat transfer coefficient is defined in the usual units of $W/(m^2 \cdot\ ^{\circ}\mathrm{C})$. The natural convective wattage for our motor cylinder is then $h_{conv} \cdot A \cdot \Delta T$ where "A" is the *total* surface area of the motor cylinder, $0.127052\, m^2$. The heat wattage transfer due to natural convection from the motor is then estimated to be $4.5590\, W/(m^2 \cdot\ ^{\circ}\mathrm{C}) \cdot 0.127052\, m^2 \cdot 22.77\ ^{\circ}\mathrm{C} = 13.19\, Watts$. This is indeed similar in magnitude to the $Q_{rad} = 15.89\, watts$ of radiated heat we calculated above. So we have accounted for $15.89\, Watts + 13.19\, Watts = 29\, Watts$ of the $83\, Watt$ heat dissipation of the motor, or 35% of that heat.

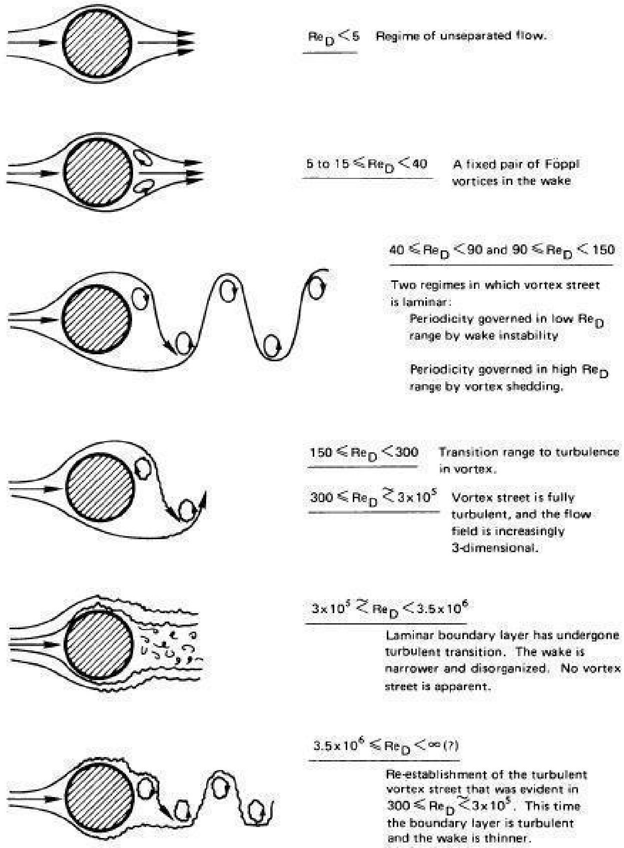
Forced convection can generally increase the heat transfer coefficient by a factor of 10 over the natural convective rate. This motor has an internal rotor fin wafting mechanism which in effect adds another forced convection path, but we will consider the contribution of that wafting mechanism later.

What would be a rough estimate of the forced convection wattage transferred from the motor to the cool air flow across its body within the air conditioner enclosure? We will consider the motor to be a cylinder with the incoming cooled air flowing transversely across it (we will also consider the air flow across the two flat end plates). The following side view of a typical evaporative air conditioner shows that the placement of the motor is within the upper area of the adjacent cooling pad through which air is drawn into the enclosure, i.e., some of the incoming air constitutes a transverse flow (though the actual flow is probably far more complex, air passing through the pad more or less perpendicular to the axis of the motor then being drawn down and to either side of the blower housing where the air rushes into the low pressure area developed by the impeller).



Because the motor casing is metallic and its size is small in relation to the air flow we will assume its Biot number is << 1, that is, the motor body surface can be regarded as a uniform temperature (rather than a gradient). The Biot number is $\bar{h}L/k_b$ where $\bar{h}$ is the convective heat transfer coefficient, k_b the thermal conductivity of the body and L is the length over which the flow passes; we will double check this after we calculate $\bar{h}$.

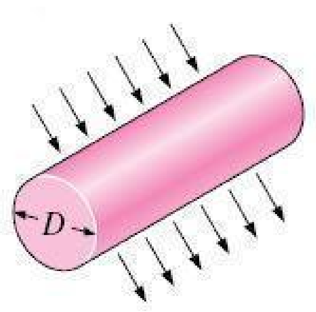
The general concept here is that fluid flowing past a warm body encounters friction and forms a thin lower velocity region or *flow boundary layer* at the body surface. Heat from the body will be conducted into this boundary layer, forming a *thermal boundary layer* (near the hot body, the cool flowing fluid soon reaches the temperature of the hot surface; that temperature gradient defines the thermal boundary layer) and is ultimately mixed and swept downstream. The forced air flow over the motor cylinder will be an *external flow* (rather than fluid flow *within* a tube or other passage). The flow in the boundary layer can be *laminar* (smooth and streamlined) or *turbulent* ("intense eddy currents and random motions of chunks of fluid," quoting from "Chapter 16... Heat and Mass Transfer," cited earlier. If the flow is *laminar* you basically see many adjacent streamlines or paths of flow progressively faster as you move away perpendicular to the surface. These streams are basically adiabatic, i.e., heat from the hot surface below the flow cannot easily cross these layered flows, reducing the conduction of heat away from the hot surface. With turbulence there is more mixing of the layers, increasing transport of the heat away from the surface. However, if the surface is fairly smooth (like a motor casing), then there is still a thin laminar layer where thermal resistance is primarily dependent on molecular diffusion rather than vertical mixing via turbulent vortices. Much work has been done in this area and we fortunately therefore have usable equations to apply (the derivation of those is fairly complex and dealt with in "A Heat Transfer Textbook" cited earlier). Fluid flow transverse to a cylinder includes different flow regimes, which are sketched in the following illustration (these regimes are implicitly incorporated in the equations we will utilize):



To begin with we will need a dimensionless Reynolds number (note the Re_D term in the flow illustration above) which "characterizes the relative influences of inertial and viscous forces in a fluid problem," put simply for our context, the Reynolds number tells us that if the fluid velocity is great and/or its viscosity low, then the thickness of the flow/velocity boundary layer will be relatively small and heat transfer will be relatively high (and conversely if the velocity is low then the boundary will be relatively thick and stagnant with lower heat transfer):

$$Re = \frac{V \cdot L_c}{v}$$

where V is the free-stream velocity of the fluid (air here) in meters per second m/s, L_c is the characteristic length of the relevant geometry, i.e., the length the fluid flows over externally ("D", the diameter of the cylinder in the figure below illustrating transverse external flow), and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid in square meters per second m^2/s. We will look up the kinematic viscosity value in a table for air at sea level pressure and the film temperature of the fluid (air) flow, $T_{film} = \frac{1}{2} \cdot (T_{body} + T_{fluid}) = \frac{1}{2} \cdot (37.77\ ^\circ\mathrm{C} + 15\ ^\circ\mathrm{C}) = 26.39\ ^\circ\mathrm{C}$ (in "A Heat Transfer Textbook" cited earlier).



We need to know the approximate velocity of the air flowing over the motor. The CFM (cubic feet per minute) of a centrifugal blower is directly proportional to the RPM of the driving motor (see Cincinnati Fan Engineering document https://www.cincinnatifan.com/catalogs/EngData-203-internet.pdf for fan laws). The nominal output specified for this air conditioner at *high speed* (1750 RPM) with 0.1 inches static pressure (static pressure head is the resistance offered to the output of the fan, e.g., high if no windows or doors open, lower if window open) is 3430 CFM (see specifications for the Champion 4001DD or Essick Air N43/48D at https://www.essickair.com/residential-aspen-media). In this calculation the motor is operating in *low* speed (1140 RPM), so the reduction in output will be a factor of $\frac{1140}{1725} = 0.66$. The nominal CFM output of this enclosure at low speed setting is therefore $0.66 \cdot 3430 = 2267\, CFM$. Multiply CFM by 0.0283 to obtain cubic meters per minute, $2267 \cdot 0.0283 = 64.19\, m^3/min$. Convert to flow per seconds by $64.19\, \frac{m^3}{min} \cdot \frac{min}{60\ sec} = 1.0699\, m^3/sec$. We will assume that an equal volume of air must enter the enclosure to produce that output.

The motor is mounted transverse to the incoming air flow from the adjacent cooling pad. The enclosure is approximately a 0.8636 meter cube with louvered input grills of 0.5058 square meter *area* on *each* of *four* sides (and each contains a fiber material or pad saturated by pump to provide evaporative cooling as the air passes through). [Using dimensions available at the Champion/Essick specifications document cited above.] The incoming air from one input grill is therefore $\frac{1}{4}$ of the output of the enclosure, i.e., $\frac{1}{4} \cdot 1.0699\, m^3/sec = 0.2673\, m^3/sec$. The velocity of the air flow over the motor is then the volumetric flow divided by the area of the adjacent grill so, $V = \frac{0.2673\, m^3/sec}{0.5058\, m^2} = 0.5285\, m/sec$.

We now return to our calculation of the forced convection flow transverse to the motor cylinder. The characteristic length of transverse flow over a cylinder is the diameter, so L_c is our motor diameter, 0.16002 meters. We just calculated the velocity of the forced convective flow as $V = 0.5285\, m/sec$. We obtain (from a table of thermophysical properties in “A Heat Transfer Textbook” cited earlier) the kinematic viscosity for air at the film temperature we calculated above as $26.39\ ^\circ\mathrm{C} \approx 300\, K^\circ$, $v = 1.578 \times 10^{-5}\ m^2/s$. The Reynolds number is then:

$$Re = \frac{V \cdot L_c}{v} = \frac{(0.5285\, m/sec) \cdot (0.16002\, m)}{1.578 \times 10^{-5}\ m^2/s} = 5359$$

That Reynolds number value selects an appropriate equation with which to calculate a Nusselt number as a function of the Reynolds number and another dimensionless number, the Prandtl number (the Prandtl number defines the thickness of the thermal boundary layer in relation to the flow/velocity boundary layer as a function of kinematic viscosity and thermal diffusivity), using a table in “Chapter 16... Heat and Mass Transfer,” cited above:

$$Nu = 0.193 \cdot Re^{0.618} \cdot Pr^{1/3}$$

The selected Nusselt equation allows us to characterize the heat transfer in cross flow over a cylinder, implicitly taking into account the complex regimes of fluid flow illustrated earlier. The forced convective heat transfer from the motor surface is then described by the relation:

$$h = \frac{k}{L_c} \cdot Nu$$

where k is the thermal conductivity of the *fluid* (air). [For a detailed discussion see “A Heat Transfer Textbook” cited earlier.] We look up the Prandtl number for the specified material, temperature and pressure, but it is around 0.7 for air at room temperature. Using a thermophysical properties table (in “A Heat Transfer Textbook”) we find $Pr = 0.713$ for the air at our previously calculated film temperature of $26.39\ ^\circ\mathrm{C} \approx 300\, K^\circ$. Plugging these values into our Nusselt number relation from above:

$$Nu = 0.193 \cdot Re^{0.618} \cdot Pr^{1/3} = 0.193 \cdot 5359^{0.618} \cdot 0.713^{1/3} = 34.768$$

As a check on the validity of this Nusselt equation I also calculated the Nusselt number with the Churchill-Bernstein equation:

$$\overline{\mathrm{Nu}}_D = 0.3 + \frac{0.62\, \mathrm{Re}_D^{1/2}\, \mathrm{Pr}^{1/3}}{\left[1 + (0.4/\mathrm{Pr})^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

I obtained a Nusselt number of 38.29 using the above Churchill-Bernstein equation, which is within 10% of the first result (so we will simply use our original result).

Looking up our k thermal conductivity value for air at our film temperature 300 K° we obtain $k = 0.02623\, W/(m \cdot K)$. Plugging in our results in the equation for the forced convection heat transfer coefficient we obtain:

$$h = \frac{k}{L_c} \cdot Nu = \frac{0.02623\, W/(m \cdot K)}{0.160020\, m} \cdot 34.768 = 5.7\, W/m^2 \cdot C$$

Note that magnitude of a degree is identical in both the Celsius and Kelvin scales so it is not an error to display our forced convection heat transfer coefficient result per degrees Celsius though the conductivity was given in degrees Kelvin.

Before we proceed to calculate the heat transfer wattage, we make a quick check on the Biot number to verify our assumption earlier that we can regard the motor as a relatively uniform hot body in a cool flow. Recall that the Biot number is $\bar{h}L/k_b$ where $\bar{h}$ is the convective heat transfer coefficient, k_b the thermal conductivity of the body and L is the length over which the flow passes. Using the calculated $\bar{h} = 5.7\, W/m^2 \cdot C$, $0.16002\, m$ for L (the transverse diameter over which the air flows), and $k_b = 43\, W/m \cdot K$ (table value for steel 1% carbon at 20 degrees C, our assumption for the metal characteristic of the motor body), we obtain $\frac{(5.7\, W/m^2 \cdot C) \cdot 0.16002\, m}{43\, W/m \cdot K} = 0.02121$. Our Biot number $0.02121 \ll 1$, which tells us the motor body surface can be regarded as a uniform temperature (rather than a gradient).

The rate of heat transfer via forced convective flow from the motor cylinder area (not including the end plates, which we will calculate shortly) is then:

$$Q = h \cdot\ A \cdot (T_{body} - T_{fluid}) = 5.7\, W/(m^2 \cdot C) \cdot 0.086829\, m^2 \cdot (37.77\ ^\circ\mathrm{C} - 15\ ^\circ\mathrm{C}) = 11.27\, W$$

We now calculate the rate of heat transfer via forced convective flow over the two end plates of the motor cylinder. The Reynolds number, $Re = \frac{V \cdot L_c}{v}$ will be the same as already calculated, 5359, since the flat plate characteristic length L_c will again be simply the motor diameter value, $0.16002\, m$, the velocity of the forced convective flow is still $V = 0.5285\, m/sec$, with identical kinematic viscosity for air at $26.39\ ^\circ\mathrm{C} \approx 300\, K^\circ$, $v = 1.578 \times 10^{-5}\ m^2/s$. That Reynolds number of 5359 gives us the corresponding relation with which to calculate the Nusselt number for the flat plate in vertical flow (using a table in “Chapter 16... Heat and Mass Transfer,” cited above):

$$Nu = 0.664 \cdot Re^{1/2} \cdot Pr^{1/3}$$

The Prandtl number is also the same, $Pr = 0.713$. Plugging our numerical values into that equation we obtain a Nusselt number for the flat plate in forced vertical flow of:

$$Nu = 0.664 \cdot Re^{1/2} \cdot Pr^{1/3} = 0.664 \cdot 5359^{1/2} \cdot 0.713^{1/3} = 43.43$$

The k thermal conductivity value for air at our film temperature of $300\,K^{\circ}$ is, as before, $k = 0.02623\,W/(m \cdot K)$. Running these number in the equation for the forced convection heat transfer coefficient we obtain for the flat plate in forced vertical flow:

$$h = \frac{k}{L_c} \cdot Nu\ = \frac{0.02623\,W/(m \cdot K)}{0.160020\,m} \cdot 43.43 = 7.12\,W/m^2 \cdot C$$

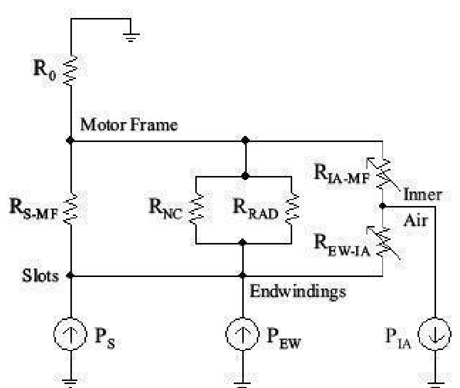
The rate of heat transfer via forced convective flow over the motor end plates (whose combined area is $0.04022\,m^2$) is then:

$$Q = h \cdot\ A \cdot (T_{body} - T_{fluid}) = 7.12\,W/(m^2 \cdot C) \cdot 0.040222\,m^2 \cdot (37.77\ ^{\circ}\mathrm{C} - 15\ ^{\circ}\mathrm{C}) = 6.52\,W$$

The total forced convective flow rate of heat transfer for the entire surface area of the motor is then the sum of the transverse cylinder rate $11.27\,W$ and the end plate rate $6.52\,W = 17.79\,W$. $17.79\,W$ is not the typical forced convection 10-fold increase over the estimated natural convective rate ($13.19\,W$). Even if we assume the forced flow assists rather than opposes the natural convection (which is reasonable since the incoming flow is more or less normal to the rising heat flow) and so permits adding the obtained results, we obtain estimated combined natural plus forced plus radiative heat transfer of $= 17.79\,W + 13.19\,W + 15.89\,W = 46.87\,W$, about 43% short of accounting for the total 83 watts of heat generated by the motor.

We need to examine the heat transfer contribution of the internal wafting mechanism to account for the remaining 43% of heat dissipation from the motor. A helpful approach will be to model our motor heat-transfer network using transfer coefficients already calculated and assigning the wafter as the unknown transfer process. Referring to source “Evolution and Modern Approaches Therm Analysis Electrical Motors,” such a network is analogous to an electrical network. The conduction, convection and radiation *thermal* resistances are calculated for the various components of the motor. The convection and radiation resistances are each equal to one divided by the product of the relevant surface area and heat transfer coefficient. Components with similar temperatures can be lumped together and represented as a single node in the thermal network. Heat transfer analysis is then the *thermal* counterpart to *electrical* network analysis with these equivalences: temperature to voltage, power to current, and thermal resistance to electrical resistance. In other words, in a thermal network a dissipation heat wattage (power) flows through a thermal resistance path and generates a temperature at each node, as analogously in an electrical network a current flows through each resistance and generates a voltage. A current flowing through a resistance creates a voltage drop, to be exact, but we just want to use the concept analogy here. One could alternately say that a temperature difference is applied to the network of thermal resistances and a resulting heat wattage flow occurs, but we are thinking of the problem more as having a known heat rate (the dissipation of the motor) and attempting to characterize the thermal resistance paths it follows to ambient.

For example, “End Space Heat Transfer Coefficient Determination for Different Induction Motor Enclosure Types” [https://www.researchgate.net/publication/224460087_End_Space_Heat_Transfer_Coefficient_Determination_for_Different_Induction_Motor_Enclosure_Types] presents a simplified thermal network representing an ODP motor (Open Drip Proof, i.e., the motor casing has small openings placed to allow heated air to escape the motor while minimizing the possibility of water entering the housing) with internal rotor fins:



The symbol P_S in the network shown represents the heat generated in the stator windings. That heat must flow through the path available through conduction, convection and radiative transfer paths between components within the motor to reach the motor frame, R_0, and through that thermal resistance to ambient air. For example, if the sum of the heat sources in the network was 100 watts and the thermal resistance of R_0, the thermal resistance of the motor frame to ambient air, was $0.3341\ ^{\circ}\mathrm{C}/watt$ then the temperature of the motor frame would be $100\,watts \cdot 0.3341\ ^{\circ}\mathrm{C}/watt = 33.41\ ^{\circ}\mathrm{C}$ above ambient.

Note the reversed heat flow generator in the schematic, P_{IA}, an “Inner Air node” power generator. The researchers added this generator to correctly model the heat removed through the motor casing vent ports by air flux generated by the rotating end-ring fins (the wafters), which otherwise would be conflated with the frame thermal resistance, R_0. Those researchers actually measured temperature at points within and without the motors under test to derive empirical models for heat transfer at different RPM. We will not be doing that here, but will accept a simplified thermal resistance equivalence at our RPM of interest (discussed below regarding R_{WAFT}). Here is an image of a disassembled typical motor with internal rotor fins marked with red arrow (to assure a concrete grasp of the description):

Let us calculate the equivalent thermal resistances of the known thermal resistance paths in our motor, i.e., R_{NCONV} (natural convection), R_{RAD} (radiation),

$R_{FORCEDENDS}$ (forced convection over endplates), and $R_{FORCEDCASE}$ (forced convection over motor case). We will solve the network constructed with these thermal resistances in parallel with the heat dissipation source (the heat dissipation of the motor calculated above, $83\,watts$) and the unknown thermal resistance equivalence of the internal rotor fin wafting process, R_{WAFT}.

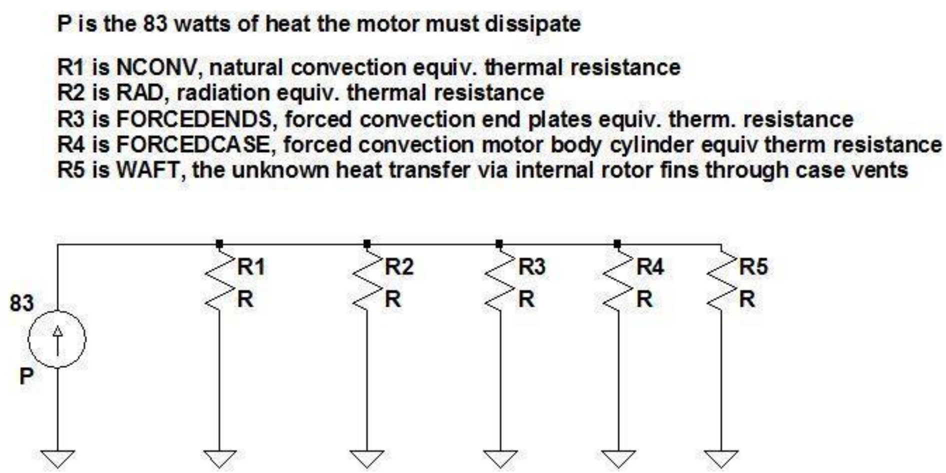
We earlier calculated the natural convection heat transfer coefficient as $4.5590\,W/(m^2\,^\circ\mathrm{C})$. That applies to the total surface area of the motor case, $0.127052\,m^2$. The equivalent thermal resistance due to natural convection is then $R_{NCONV} = \frac{1}{h \cdot A} = \frac{1}{(4.5590\,W/(m^2\,^\circ\mathrm{C}))\cdot 0.127052\,m^2} = 1.73^\circ\mathrm{C/W}$.

Using our radiation heat transfer coefficient of $h_{rad} = 5.49\,W/m^2 \cdot K$ calculated earlier, we obtain an equivalent thermal resistance due to radiation transfer from motor case to ambient of $R_{RAD} = \frac{1}{h \cdot A} = \frac{1}{(5.49\,W/(m^2\,^\circ\mathrm{C}))\cdot 0.127052\,m^2} = 1.43^\circ\mathrm{C/W}$.

Earlier we calculated the forced convection heat transfer coefficient for the motor end plate surface area as $7.12\,W/m^2 \cdot {}^\circ\mathrm{C}$. Our equivalent thermal resistance for the motor end plate surface area is then $R_{FORCEDENDS} = \frac{1}{h \cdot A} = \frac{1}{(7.12\,W/(m^2\,^\circ\mathrm{C}))\cdot 0.04022\,m^2} = 3.49^\circ\mathrm{C/W}$. Note that the area here is only the surface area of the motor end plates, not the entire surface area of the motor.

The previously calculated forced convection heat transfer coefficient for the motor cylinder surface area (excluding the flat end plates) was $h = 5.7\,W/m^2\,^\circ\mathrm{C}$. The equivalent thermal resistance for the motor cylinder surface area is then $R_{FORCEDCASE} = \frac{1}{h \cdot A} = \frac{1}{(5.7\,W/(m^2\,^\circ\mathrm{C}))\cdot 0.086829\,m^2} = 2.02^\circ\mathrm{C/W}$. Note that the area here is only the surface area of the motor body cylinder, excluding the flat end plates.

Our equivalent thermal resistance network then appears as:



Resistances in parallel sum as their reciprocals and the system dissipation (83 watts) will flow through that equivalent parallel thermal resistance to produce the motor case temperature elevation above ambient that we measured, $\Delta T = 22.77^\circ\mathrm{C}$. That is:

$$83\,Watts \cdot \frac{1}{\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} + \frac{1}{R4} + \frac{1}{R5}} = 22.77^\circ\mathrm{C}$$

Using the equivalent thermal resistances we calculated we have:

$$83\,Watts \cdot \frac{1}{\frac{1}{1.73} + \frac{1}{1.43} + \frac{1}{3.49} + \frac{1}{2.02} + \frac{1}{R5}} = 22.77^\circ\mathrm{C}$$

Adding the reciprocals and doing some basic math:

$$83 \cdot \frac{1}{2.06 + \frac{1}{R5}} = 22.77$$

$$\frac{83}{22.77} = 2.06 + \frac{1}{R5}$$

$$1.59 = \frac{1}{R5}$$

$$0.63 = R5$$

We conclude that at the specified motor RPM, efficiency and ambient temperature the internal motor fins "wafter" removing internal motor heat by blowing hot air out of the motor casing through vents has an equivalent thermal resistance of $0.63^\circ\mathrm{C/W}$. We check our result by multiplying the resultant total equivalent motor thermal resistance times the 83 watts of power it must dissipate (which should give us the measured temperature elevation above ambient):

$$83\,Watts \cdot \frac{1}{\frac{1}{1.73} + \frac{1}{1.43} + \frac{1}{3.49} + \frac{1}{2.02} + \frac{1}{0.63}} = 83\,Watts \cdot 0.27^\circ\mathrm{C/W} = 22.77^\circ\mathrm{C}$$

We can solve the equivalent system thermal resistance without the wafting process to obtain $0.49^{\circ}\mathrm{C/W}$, i.e., representing the sum of the motor natural convection, radiation and forced convective paths without the benefit of the internal motor fins and venting. As we mentioned at the beginning of this document, this is equivalent to a combined heat transfer coefficient of about $16\, watts/(m^2 \cdot {}^{\circ}\mathrm{C})$, very close to industry standard range of $12 - 14\, watts/(m^2 \cdot {}^{\circ}\mathrm{C})$.

Solving the equivalent system thermal resistance with the wafter thermal resistance of $0.63^{\circ}\mathrm{C/W}$ gives an equivalent total system thermal resistance of $0.27^{\circ}\mathrm{C/W}$ (which we tested by multiplying times the system heat wattage of 83 above to obtain the measured temperature elevation above ambient of $22.77^{\circ}\mathrm{C}$). The reduction of system thermal resistance by the internal wafting is then about $\frac{0.27}{0.49} = 56\,\%$. For comparison, in "End Space Heat Transfer Coefficient Determination for Different Induction Motor Enclosure Types" a 5 HP DP motor (a large motor with internal rotor fins and vents) dropped from a frame to ambient equivalent thermal resistance of $0.2216^{\circ}\mathrm{C/W}$ at 0 rpm to 0.075 at 1000 rpm, or 34% of original frame resistance (note that the higher horsepower motor has larger surface area and lower basic thermal resistance; this 5 HP motor was measured with a constant injection of current and external rotation of a substitute dummy rotor with fins; see article for details). We are dropping from $0.49^{\circ}\mathrm{C/W}$ to $0.27^{\circ}\mathrm{C/W}$, or $56\,\%$ of base thermal resistance with internal wafting at 1140 RPM. It should be noted that our basic thermal resistance value (our "0 RPM" value) includes the relatively weak 1140 rpm forced convective cooling provided by the incoming air in the AC housing (calculated earlier), whereas the 5 HP motor 0 RPM value was an empirical measurement without any forced convection, but we are just making a rough comparison to satisfy ourselves that we have correctly estimated the various heat transfer processes in our motor. We could attempt a rough model of the internal rotor fin as a vane axial fan (to estimate the internal flow and estimated heat removal), using the little we can infer or guess about the number of blades, their pitch, and the fan tip diameter, but it appears likely that our estimated internal wafter heat removal is in the ballpark of what is typical, making further approximations unnecessary.